

Non-compact QED₃ at finite temperature: the confinement-deconfinement transition

Roberto Fiore,^a Pietro Giudice^b and Alessandro Papa^a

^a*Dip. di Fisica, Università della Calabria and INFN, Gruppo collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy*

^b*Dip. di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy*

E-mail: fiore@cs.infn.it, giudice@to.infn.it, papa@cs.infn.it

ABSTRACT: The confinement-deconfinement phase transition is explored by lattice numerical simulations in non-compact (2+1)-dimensional quantum electrodynamics with massive fermions at finite temperature. The existence of two phases, one with and the other without confinement of fractional charges, is related to the realization of the \mathbb{Z} symmetry. The order parameter of this transition can be clearly identified. We show that it is possible to detect the critical temperature for a given value of the fermion mass, by exploiting suitable lattice operators as probes of the \mathbb{Z} symmetry. Moreover, the large-distance behavior of the correlation of these operators permits to distinguish the phase with Coulomb-confinement from the Debye-screened phase. The resulting scenario is compatible with the existence of a Berezinsky-Kosterlitz-Thouless transition. Some investigations are presented on the possible relation between chiral and deconfinement transitions and on the role of “monopoles”.

KEYWORDS: Confinement, Spontaneous Symmetry Breaking, Lattice Gauge Field Theories, Global Symmetries.

Contents

1. Introduction	1
2. The continuum QED₃ theory	2
2.1 The \mathbb{Z} symmetry	3
2.2 Theoretical expectation for the phase diagram	4
3. The lattice QED₃ theory	5
3.1 Order parameters and other lattice operators	6
3.2 General strategy of the lattice calculation	7
4. Numerical results	7
4.1 The algorithm	7
4.2 Order parameter	8
4.3 Polyakov loop correlators	10
4.4 Chiral condensate	19
4.5 Monopole density	21
5. Conclusions and outlook	22

1. Introduction

Non-compact quantum electrodynamics in (2+1)-dimensions (QED₃) with fermionic matter is a special theory since it plays a role in contexts ranging from condensed matter to particle physics. In particular, this theory is relevant for the characterization of the phase diagram of high- T_c superconductors in the temperature-doping plane (see, for instance, ref. [1] and references therein). Moreover, it is an interesting theoretical laboratory for the investigation of mechanisms of confinement and for the study of the confinement-deconfinement transition at finite temperature.

This paper aims at contributing to clarify some issues related with the latter topic. Understanding the mechanism of confinement, identifying the nature of the confinement-deconfinement transition and the related order parameter is of central interest in finite-temperature non-Abelian quantum field theories, such as Quantum Chromodynamics (QCD). Here definite answers are available in the limiting cases of infinite quark masses (i.e. in the pure gauge theory) and zero quark masses (i.e. in the chiral limit). Instead, many open questions remain for the general case of fermions with finite non-zero mass. In non-compact QED₃ the symmetry whose breaking determines the transition is the \mathbb{Z} symmetry, independently from the fermionic mass, the order parameter being the Polyakov loop with

fractional charge. Evaluating the vacuum expectation value of the order parameter and its 2-point correlator allows to get, at least in principle, a complete description of the phase diagram of the theory for varying fermion masses, coupling constant and temperature.

The only theoretical scenario suggested so far for the phase diagram of non-compact QED₃ at finite temperature is presented in refs. [2, 3], where it is found that the effective action for the temporal component of the gauge field, $A_0(x)$, in the limit of large fermion mass becomes equivalent to the sine-Gordon potential. This led the authors of ref. [2] to conclude that there is a phase transition of Berezinsky-Kosterlitz-Thouless (BKT) type [4, 5]. More precisely, there is a critical temperature T_c , depending on the fermion mass, above which the system is in a deconfined or Debye phase, while below T_c the interaction is logarithmic with the distance, i.e. it is Coulomb-like. In this paper we intend to verify the above scenario for non-compact QED₃ at finite temperature, by Monte Carlo numerical simulations, using as probes the Polyakov loop with fractional charge and its 2-point correlator.

Another topic we consider in this paper is the behavior of the chiral condensate across the deconfinement transition. In particular, if the chiral condensate exhibits a sharp drop when the temperature is increased through the critical value, then we can conclude that the dynamical mechanism responsible for deconfinement affects also the chiral symmetry of the theory.

Finally, we present a few numerical results concerning the magnetic monopole density. While monopoles in compact QED₃ are undoubtedly related to confinement in the pure gauge case [6] and the same is argued also in presence of fermions [7–9], in the non-compact theory there is no *a priori* reason for which they could play any role. However, following ref. [10], a monopole density can be defined on the lattice in the non-compact theory exactly as in the compact one and its behavior with temperature can be studied. In (3+1)-dimensional non-compact QED, it turns out that there exist a percolation threshold for monopole current networks near the chiral transition [10], thus suggesting a possible relation between monopole condensation and chiral symmetry breaking (see ref. [11] for a criticism to this approach). In this paper we present some determinations for the monopole density across the deconfinement transition and briefly discuss the possibility of a relation between monopoles and confinement in non-compact QED₃.

The paper is organized as follows: in section 2 we briefly recall the theory in the continuum formulation, the origin of the \mathbb{Z} symmetry and the conjectured phase diagram; in section 3 we describe the lattice version of the theory and the operators we use as probes of the phase transition; in section 4 we present the numerical results and discuss their interpretation; in section 5 we draw our conclusions and sketch the future perspectives.

2. The continuum QED₃ theory

The continuum Lagrangian density describing QED₃ is given in Minkowski metric by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}_i i D_\mu \gamma^\mu \psi_i - m_0 \bar{\psi}_i \psi_i, \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength, $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative, g is the coupling constant (or the electric charge) and the fermion fields ψ_i ($i = 1, \dots, N_f$)

are 4-component spinors, defined in such a way that the theory is parity-invariant.¹ The relevant information concerning parity and chiral symmetry of this model can be found, for instance, in ref. [9], section II.

2.1 The \mathbb{Z} symmetry

In this section we briefly recall the origin of the \mathbb{Z} symmetry which plays a fundamental role in this paper.

The Euclidean partition function of finite temperature QED is invariant under gauge transformations [2, 3]. For the field $A_\mu(\tau, \vec{x})$ the gauge transformation

$$A'_\mu(\tau, \vec{x}) = A_\mu(\tau, \vec{x}) + \partial_\mu \chi(\tau, \vec{x}) , \quad (2.2)$$

together with the periodic boundary conditions in the time direction

$$A_\mu(1/T, \vec{x}) = A_\mu(0, \vec{x}) , \quad (2.3)$$

where T is the temperature, implies the following condition:

$$\partial_\mu \chi(1/T, \vec{x}) = \partial_\mu \chi(0, \vec{x}) . \quad (2.4)$$

Alike, for the fermion field $\psi(\tau, \vec{x})$, the gauge transformation

$$\psi'(\tau, \vec{x}) = e^{ig\chi(\tau, \vec{x})} \psi(\tau, \vec{x}) , \quad (2.5)$$

together with antiperiodic boundary conditions in the time direction

$$\psi(1/T, \vec{x}) = -\psi(0, \vec{x}) , \quad (2.6)$$

implies

$$\chi(1/T, \vec{x}) = \chi(0, \vec{x}) + \frac{2\pi}{g} n , \quad (2.7)$$

where n is an integer. Differently, we can say that $\chi(x)$ is periodic in the time direction with period $1/T$ up to an integer multiple of $2\pi/g$. In the language of group theory, we can say that if G is the group of all gauge transformations and H is the subgroup of those gauge transformations which are strictly periodic, then the quotient group G/H is isomorphic to \mathbb{Z} , the additive group of integers. We refer to this when we say that the theory possesses \mathbb{Z} symmetry. Note that this is a symmetry of the partition function *in presence of dynamical electrons*.

In order to study the realization of the \mathbb{Z} symmetry, a version of the Polyakov loop operator is introduced, whose average is related to the free energy of an external charge \tilde{g} :

$$\Pi_{\tilde{g}}(\vec{x}) = e^{i\tilde{g} \int_0^{1/T} dx_0 A_0(x_0, \vec{x})} . \quad (2.8)$$

Under the action of an element of G/H , the Polyakov loop operator with charge \tilde{g} transforms as (here we use eqs. (2.2) and (2.7))

$$\Pi'_{\tilde{g}}(\vec{x}) = e^{i\tilde{g} \int_0^{1/T} d\tau A'_0(\tau, \vec{x})} = e^{i\tilde{g} \int_0^{1/T} d\tau A_0(\tau, \vec{x})} e^{\frac{i2\pi n \tilde{g}}{g}} = \Pi_{\tilde{g}}(\vec{x}) e^{\frac{i2\pi n \tilde{g}}{g}} . \quad (2.9)$$

¹This implies that the fermion is massive, while the photon is massless.

Therefore, in order that the operator defined in eq. (2.8) be an order parameter, the charge \tilde{g} must not be an integer multiple of the basic charge g . One possibility is to follow ref. [12] and study charges that are rational fractions of the fundamental charge, i.e. $\tilde{g} = g/m$, with integer m . As a consequence, eq. (2.9) becomes

$$\Pi'_m(\vec{x}) = \Pi_m(\vec{x}) e^{\frac{i2\pi n}{m}}. \quad (2.10)$$

The realization of the \mathbb{Z} symmetry tests the ability of the electrodynamic system to screen charges which are not integral multiples of the electron charge. Therefore, it is related to (fractional) charge screening and confinement in QED in the same way as the \mathbb{Z}_N symmetry does in finite temperature $SU(N)$ pure gauge theory.

In conclusion, the operator $\Pi_{\tilde{g}}(\vec{x})$ can be used as an order parameter for confinement in Abelian gauge theories even in presence of dynamical charged particles: if the symmetry is unbroken, the loop operator averages to zero and the system is in the confining phase, otherwise it is in a non-confining phase.

2.2 Theoretical expectation for the phase diagram

In this Subsection, we briefly recall the ideas put forward in ref. [2] about the phase structure of the theory under consideration.

The authors of ref. [2] propose the Polyakov loop with fractional charge as order parameter and argue that in parity-invariant electrodynamics with fermions of mass M there is a deconfinement transition at finite temperature of BKT type.

The analysis of ref. [2] is based on computing the effective action $V(M, gA_0/T)$ for $A_0(\tau, \vec{x})$, which explicitly exhibits the global \mathbb{Z} symmetry. By studying this effective action they characterize the type of the phase transition.

At finite temperature, non-compact QED_3 contains three parameters (with the dimension of mass): the fermion mass M , the gauge coupling g^2 and the temperature T ; the dimensionless parameter which governs the loop expansion is the smaller between g^2/M and g^2/T .

In the large M limit, so that T/M and g^2/M are small with g^2/T finite, they argue that the critical behavior of the theory is identical to that of the 2-dimensional sine-Gordon potential, which undergoes a BKT phase transition. Therefore they conclude that also in QED_3 at finite temperature there must be a BKT transition with a critical line in the $[M/T, g^2/T]$ plane, starting at

$$(M/T, g^2/T) = (\infty, 8\pi), \quad (2.11)$$

from which they obtain

$$T_{crit.}^{M \gg T} = \frac{g^2}{8\pi}. \quad (2.12)$$

They determine also the critical temperature for the BKT transition up to one-loop order:

$$T_{crit.}^{M \gg T} = \frac{g^2}{8\pi(1 + \frac{g^2}{12\pi M})}. \quad (2.13)$$

The second limit considered in ref. [2] is the high-temperature limit, $T \gg M, g^2$. The analysis of ref. [2] shows that the \mathbb{Z} symmetry is spontaneously broken, which means the system is in the deconfined phase.

In a subsequent paper [3] the same authors characterize better the nature of the two phases divided by the BKT critical line. They find that, where the \mathbb{Z} symmetry is unbroken, the system is in a confining phase, more precisely in a ‘‘Coulomb phase’’, where electric charges mutually interact by a logarithmic Coulomb potential. This logarithmic behavior has as a consequence a power law dependence of the Polyakov loop correlators. Indeed, if the interaction potential is of the form $V(r) = \alpha \log r$, then one can write immediately

$$G(r) = \langle \Pi(0)\Pi^*(r) \rangle = e^{-\frac{V(r)}{T}} = e^{-\frac{\alpha}{T} \log r} = r^{-\frac{\alpha}{T}} = r^{-\eta(T)}. \quad (2.14)$$

At the tree level, they find

$$G(r)_{\text{tree}} \propto r^{-\frac{\tilde{g}^2}{4\pi T}}. \quad (2.15)$$

From eqs. (2.14) and (2.15) the value of η can be easily found:

$$\eta = \frac{\tilde{g}^2}{4\pi T}. \quad (2.16)$$

Above T_c the electric charge is not confined and the system is in a Debye plasma phase. The expected *large distance* behavior for the *connected* Polyakov loop correlators in this case is given by

$$G_{\text{conn}}(r) \propto e^{-M_D r}. \quad (2.17)$$

The Debye mass M_D makes the Coulomb interaction short-ranged and fermions and antifermions are approximately free particles.

3. The lattice QED₃ theory

In this paper we discretize the Euclidean action on a lattice with $N_\sigma^2 \times N_\tau$ sites and lattice spacing a by staggered fermions $\bar{\chi}, \chi$, according to

$$S = S_G + \sum_{i=1}^N \sum_{n,m} \bar{\chi}_i(n) M_{n,m} \chi_i(m), \quad (3.1)$$

where S_G is the gauge field action and the fermion matrix is given by

$$M_{n,m}[U] = \sum_{\nu=1,2,3} \frac{\eta_\nu(n)}{2} \left\{ [U_\nu(n)] \delta_{m,n+\hat{\nu}} - [U_\nu^\dagger(m)] \delta_{m,n-\hat{\nu}} \right\}, \quad (3.2)$$

with $\eta_\nu(n) = (-1)^{\sum_{\rho<\nu} n_\rho}$ and $U_\mu(n) = e^{i\alpha_\mu(n)}$ is the link variable; the phase α is related to the gauge field by $\alpha_\mu(n) = agA_\mu(n)$.

In a non-compact formulation of QED₃, S_G is given by

$$S_G[\alpha] = \frac{\beta}{2} \sum_{n,\mu<\nu} F_{\mu\nu}(n) F_{\mu\nu}(n), \quad (3.3)$$

where

$$F_{\mu\nu}(n) = \{\alpha_\nu(n + \hat{\mu}) - \alpha_\nu(n)\} - \{\alpha_\mu(n + \hat{\nu}) - \alpha_\mu(n)\} \quad (3.4)$$

and $\alpha_\mu(n)$ is the phase of the link variable and $\beta = 1/(g^2 a)$.

With this action we simulate $N = 1$ flavors of staggered fermions corresponding to $N_f = 2$ flavors of parity-invariant four-component fermions.

3.1 Order parameters and other lattice operators

The lattice version of the operator (2.8) is [12]

$$\Pi_m(\vec{x}) = \prod_{\tau=1}^{N_\tau} e^{\frac{i}{m}\theta(\vec{x},\tau)}, \quad (3.5)$$

where $\theta = gA_0$ and \vec{x} lives on the N_σ^2 spatial lattice, from which we can determine the operator averaged on the lattice configuration,

$$\Pi_m = \frac{1}{N_\sigma^2} \sum_{\vec{x}} \Pi_m(\vec{x}). \quad (3.6)$$

As noted in ref. [12], the original \mathbb{Z} symmetry is translated into a \mathbb{Z}_m symmetry on the lattice operator Π_m ; in the broken phase, the operator Π_m will fluctuate around the values $e^{i\frac{2\pi}{m}k}$, where $k = 0, 1, \dots, m - 1$. In order to study the breaking of this symmetry it is useful therefore to introduce the following operator:

$$\Pi_m^m = \left(\frac{1}{N_\sigma^2} \sum_{\vec{x}} \Pi_m(\vec{x}) \right)^m. \quad (3.7)$$

Another possible choice of lattice order parameter is [13]

$$\Theta = \cos [m \times \arg(\Pi_m)] ; \quad (3.8)$$

both definitions (3.7) and (3.8) have the effect to “rotate” the m different phases to the direction corresponding to $\theta = 0$.

There are two other quantities of physical relevance to be considered; one is the monopole density ρ , which can be defined in the same way as in the compact theory, using the method of ref. [14],

$$\rho = \frac{1}{2} \frac{\langle N_M \rangle + \langle N_{\bar{M}} \rangle}{N_\sigma^2 N_\tau}, \quad (3.9)$$

where N_M ($N_{\bar{M}}$) is the number of monopoles (antimonopoles); the other is chiral condensate, $\langle \bar{\chi}\chi \rangle$.

The presence of transitions is detected by looking for peaks in the susceptibility of the operators (3.6), (3.7) and (3.8), the susceptibility of a generic operator \mathcal{O} being defined as

$$\chi_{\mathcal{O}} = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2. \quad (3.10)$$

The susceptibilities of monopole density and chiral condensate is also considered in order to study the possible relation of these latter operators with the confinement/deconfinement transition.

3.2 General strategy of the lattice calculation

The main goal of the numerical computations in this paper is to find evidences of the existence of a transition between two phases, one with unbroken and the other with broken \mathbb{Z} symmetry. The strategy for that is to scan the temperature for a fixed value of the bare fermion mass aM and on lattices with given extension to study the behavior of the lattice order parameters defined in the previous subsection and of their susceptibility. On the lattice, we have

$$T = \frac{1}{N_\tau a} = \frac{g^2}{N_\tau} \beta, \quad (3.11)$$

therefore at fixed coupling constant g and on a lattice with fixed N_τ , the temperature can be changed by changing β (note that the theory is super-renormalizable, therefore g does not depend on a). We expect that changing β we meet a clear signature of the transition, in correspondence to the critical temperature, which, at infinite fermion mass, is given in eq. (2.12) and, through (3.11), can be translated on the lattice to the following critical value for the β parameter:

$$\beta_c(M \rightarrow \infty) = \frac{N_\tau}{8\pi}. \quad (3.12)$$

The next step is to study the behavior of the Polyakov loop correlator. If the theoretical expectations introduced in section 2.2 are true, then for a β value below the critical one, $\beta_c(M)$, we should find a power law behavior, whereas above $\beta_c(M)$ there should be exponential fall-off. In the Coulomb-confined phase, i.e. for $\beta < \beta_c(M)$, where the power law behavior is expected, we can compare our findings with the expected tree-level value of η given in eq. (2.16), which in lattice units reads

$$\eta = \frac{N_\tau}{m^2 4\pi \beta}. \quad (3.13)$$

We recall that $1/m$ represents the ratio \tilde{g}/g , i.e. the fraction charge in units of the fundamental charge. It should be noted here that η is proportional to $1/m^2$.

4. Numerical results

In this section we present our numerical results obtained on lattices $12^2 \times 8$, $32^2 \times 8$ and $64^2 \times 8$. On the smallest lattice considered, we found that tunneling effects among the different \mathbb{Z}_m vacua of the theory in the broken symmetry phase are evident in the expectation value of Π_m . For this observable this makes possible to better unravel the vacuum structure in the deconfined phase. However, for the study of correlation functions, where tunneling in the deconfined phase is potentially dangerous, we used lattices with $N_\sigma = 32$ and 64 for which we verified that tunneling is absent.

4.1 The algorithm

We used the Hybrid Monte Carlo Algorithm [15], built by superimposing the Metropolis acceptance test to the Hybrid Algorithm in the structure proposed in ref. [16]). Here is a list of the internal parameters of the Monte Carlo code:

- the mass term ω for the pseudofermionic field in the Hamiltonian, which drives the molecular dynamics;
- the integration step δt , fixed to 0.03 in all simulations;
- the frequency MCR of the algorithm refreshments and Metropolis tests.

We have verified the important role of ω in simulations: if ω is too small ($\omega \ll 1$), the system thermalizes very slowly, but observables do not fluctuate much around the mean value during the simulation time; on the contrary, if ω is not so small ($\omega \lesssim 1$), the system thermalizes very quickly with the disadvantage that fluctuations increase considerably. It is therefore fundamental to optimize the choice of the parameter ω .

Additionally, we use the following abbreviations: FOM for the frequency of measurements and NMS for the number of measurements or statistics. The bare fermion mass is fixed to $aM = 0.05$.

4.2 Order parameter

In figure 1 we show scatter plots of the order parameter Π_m in the complex plane for a β value known *a posteriori* to lie in the confined phase and for several values of m . For $m = 1, 2$ data distribute on a disk, while for $m \geq 3$ they are on a ring whose radius increases with m ; in both cases data are uniformly spread around the origin. Figure 2 gives the evolution in the simulation time of Π_{10} ; it is interesting to observe that $\arg \Pi_{10}$ spans in a continuous way the interval $[0, 2\pi]$ (with account of the 2π periodicity).

Figure 3 is the same as figure 1, but for a value of β in the deconfined phase. In this case the \mathbb{Z}_m symmetry is broken and the values of the order parameter accumulate in correspondence of the m roots of the identity. Accordingly, the evolution in simulation time of $\arg \Pi_{10}$ spans the interval $[0, 2\pi]$ in jerky way (see figure 4).

These scatter plots tell us that real and imaginary parts of $\langle \Pi_m \rangle$ are zero in both phases, if, evidently, there is enough tunneling in the deconfined phase and if the algorithm explores the whole configuration space in the confined phase, as it seems to be the case on the lattice $12^2 \times 8$ used to obtain the above figures. So, the more informative quantity here seems to be $\langle \text{abs}(\Pi_m) \rangle$. The behavior of this quantity is shown in figure 5 for varying β and for several values of m and in figure 6 for varying m and for different values of β . Figure 5 shows that there is a smooth increase with β , more evident for higher values of m . Data do not allow to clearly single out any transition; indeed, also the susceptibility of $\text{abs}(\Pi_m)$ does not show clear peaks for varying β in the same region (see figure 7). Therefore, we can conclude that this operator is not convenient to distinguish the two phases which lead to the different behaviors shown in figures 1 and 3.

This conclusion suggests to move to the operator Π_m^m , which evidently has no effect in the angular distribution in the confined phase, i.e. in the situation of figure 1, while “concentrates” the angular distribution along the first of the m roots of the identity in the deconfined phase, i.e. in the situation of figure 3. Equivalently, we can say that $\langle \arg \Pi_m^m \rangle = 0$ in the deconfined phase. The operator Π_m^m should therefore permit to distinguish between

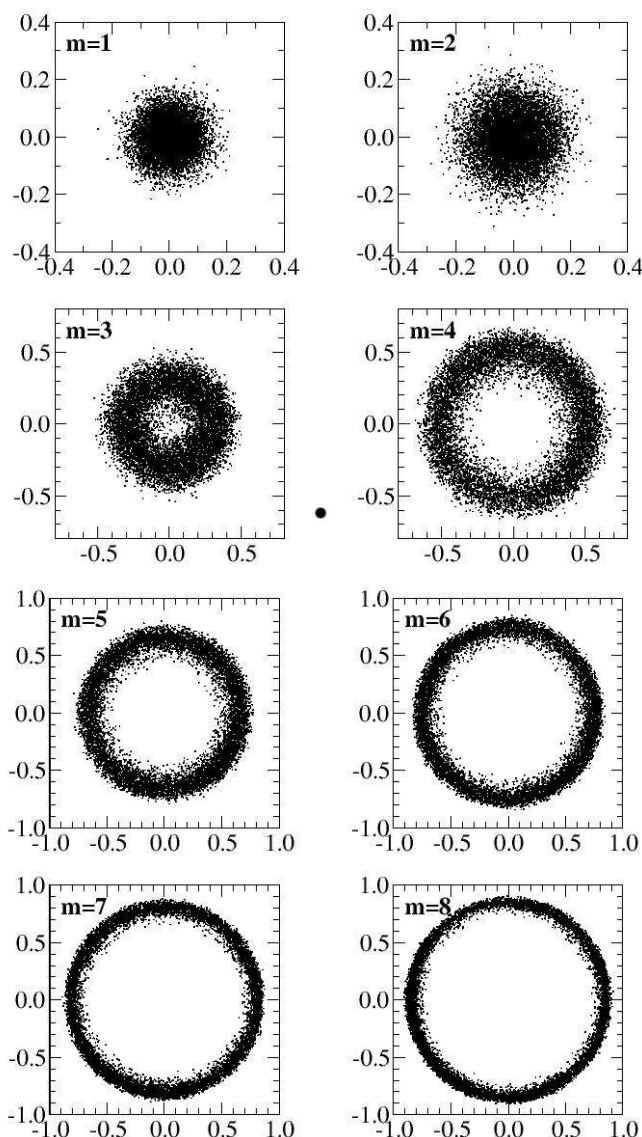


Figure 1: Scatter plots of Π_m ($\text{Im}\Pi_m$ versus $\text{Re}\Pi_m$) for 8 values of m on a $12^2 \times 8$ lattice at $\beta = 0.15$ (confined phase); the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000. For $m = 1, 2$ data are homogeneously distributed on a disc; for $m \geq 3$ data are homogeneously distributed on a ring whose radius increases with m .

broken and unbroken symmetry phases; since $\text{Im}\langle\Pi_m^m\rangle$ is always zero, we consider from the beginning $\text{Re}\langle\Pi_m^m\rangle$.

Results are shown in figures 8 and 9. In this case we have a clear hint on the position of the transition point between the two phases. In particular, figure 8 indicates that, for all values of $m > 1$, there is a sharp increase of the signal at $\beta \simeq 0.3$, while figure 9 suggests that the increase is more pronounced the higher is m . For $m = 1$ data show no transition at all, as it must be, since $m = 1$ means integer electric charge. This is confirmed in figure 10 where a peak in the susceptibility of the operator $\text{Re}\Pi_m^m$ appears for all values of $m > 1$ at

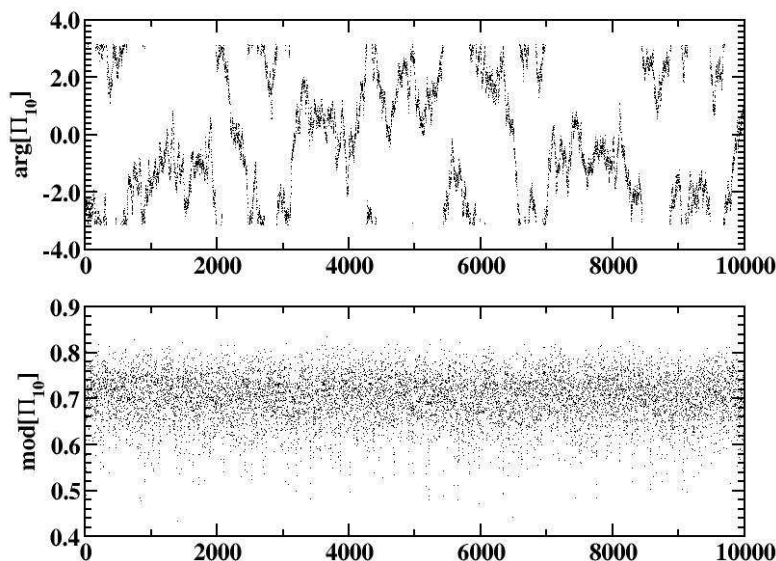


Figure 2: Evolution with Monte Carlo time of modulus (down) and argument (up) of Π_{10} on a $12^2 \times 8$ lattice at $\beta = 0.05$ (confined phase); the simulation parameters are as in figure 1.

a β_c value which, on this lattice, has a mild dependence on m , namely, it decreases with m and seems to stabilize for high m around $\beta_c = 0.32$.

In figures 11 and 12 we show the same results obtained on a lattice $32^2 \times 8$. In this case the values of β where the peaks appear are more stable with m and we can estimate the critical value as $\beta_c = 0.33$. This leads us to argue that the m -dependence of β_c seen on the smaller lattice could be a finite volume effect.

It is important to compare the numerical result for β_c with the theoretical expectation discussed in section 2.2, in particular with the value of β_c given in eq. 3.12: $\beta_c^{\text{th}} = \pi^{-1} \simeq 0.318\dots$. This value is impressively close to our numerical result, thus supporting the theoretical scenario proposed in ref. [2] and the conjectured BKT transition (see also ref. [17]).

In figures 13 and 14 we consider the observable $\langle \cos[m \times \arg \Pi_m] \rangle$, which should give similar information as the previous observable, concerning the location of the transition point. We see that data for $m > 1$ fall on top of each other, while data for $m = 1$ describe a different curve, but also exhibit a sharp increase at the same point as data for $m > 1$. Since we know that for $m = 1$ the observable cannot be an order parameter, this effect should be an artifact of the finite volume. Unfortunately, the susceptibility, shown in figure 15, does not have a peak structure, but shows rather a jump between two constant values, therefore it does not allow for an accurate determination of β_c .

4.3 Polyakov loop correlators

We have studied the wall-wall correlation between two Polyakov loops²

$$G(r) = \text{Re} \langle \Pi(0) \Pi^*(r) \rangle \tag{4.1}$$

²Note that the value of $G(r) = \text{Im} \langle \Pi(0) \Pi^*(r) \rangle$ is always compatible with zero.

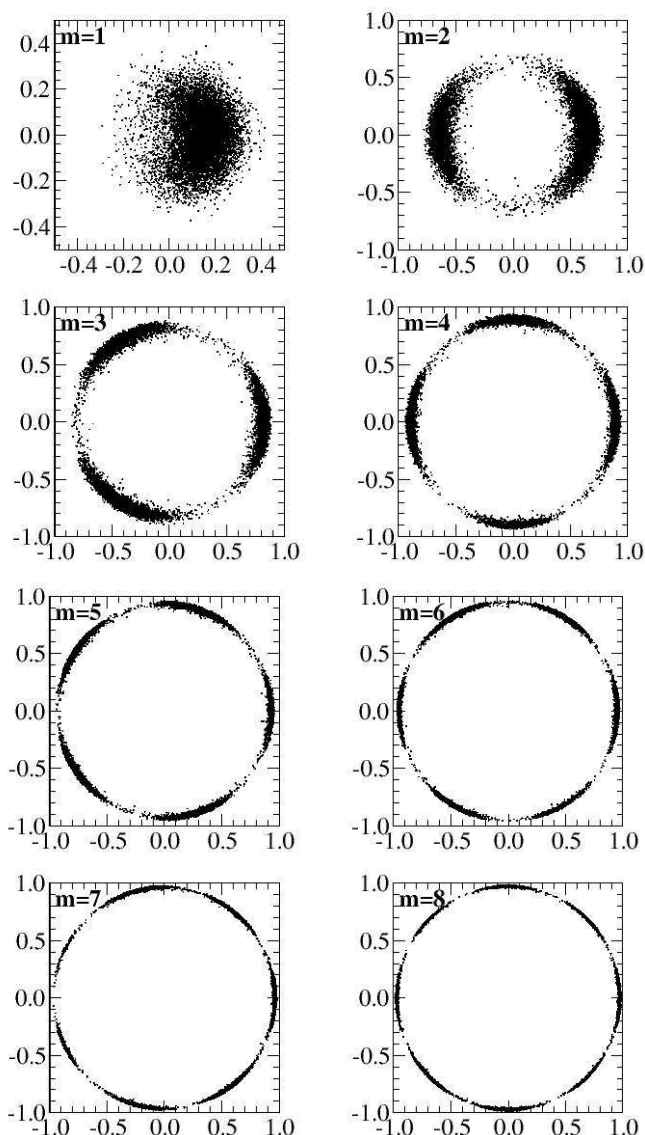


Figure 3: Scatter plots of Π_m ($\text{Im}\Pi_m$ versus $\text{Re}\Pi_m$) for 8 values of m on a $12^2 \times 8$ lattice at $\beta = 0.80$ (deconfined phase); the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000. In this case data are distributed on the m roots of unity.

for two values of β , $\beta = 0.25$ and $\beta = 0.40$, the first below the critical temperature and the other above it.

It is worth noting here that the dimensionless parameter g^2/M is equal to 80 for $\beta = 0.25$ and to 50 for $\beta = 0.40$. The other important dimensionless parameter T/M is equal to 2.5 in all simulations performed in this work. These values for g^2/M and T/M are far from the large M limit, where the theoretical analysis of refs. [2, 3] was carried on (see Subsection 2.2). However, the authors of ref. [3] state that they expect their results are stable toward lower electron masses.

For $\beta = 0.40$ we have simulated the system on two different lattices: $32^2 \times 8$ with

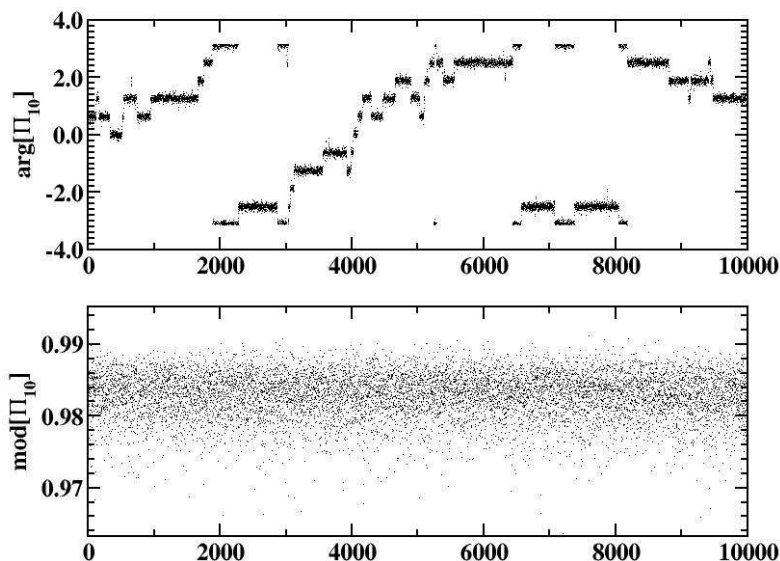


Figure 4: Evolution with Monte Carlo time of modulus (down) and argument (up) of Π_{10} on a $12^2 \times 8$ lattice at $\beta = 0.80$ (deconfined phase); the simulation parameters are as in figure 3.

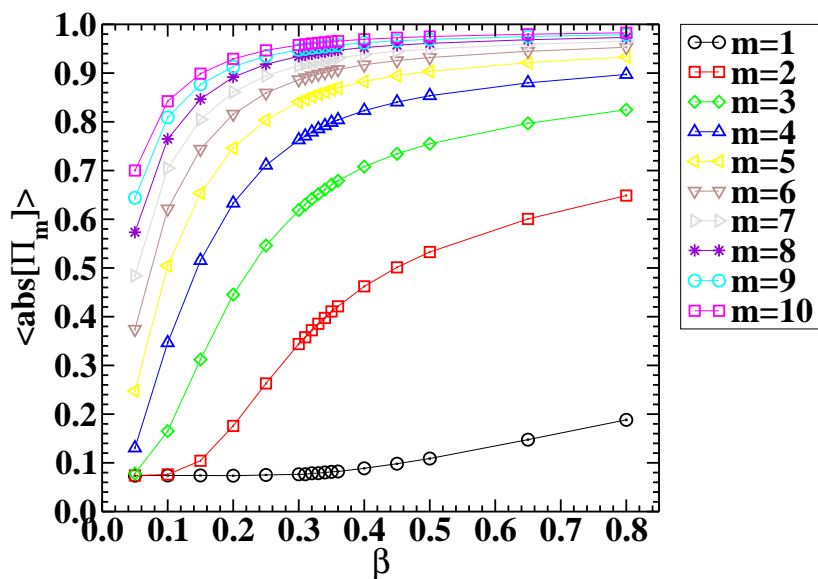


Figure 5: Modulus of $\langle \Pi_m \rangle$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, $MCR=50$, $FOM=1000$, $NMS=10000$.

$\omega = 0.2$, $MCR=10$, $FOM=10$, $NMS=100000$; $64^2 \times 8$ with $\omega = 0.1$, $MCR=50$, $FOM=250$, $NMS=8000$. Data for the correlator, shown in figure 16 (see also figure 17 for the scatter plot of Π_m) have been fitted, in a range $[r_{\min}, r_{\max}]$, using the law

$$G(r) = A \left(e^{-Mr} + e^{-M(N_\sigma - r)} \right) + C ; \quad (4.2)$$

for both lattices and for the different values of m we have obtained $\chi^2/\text{d.o.f.} < 1$. We

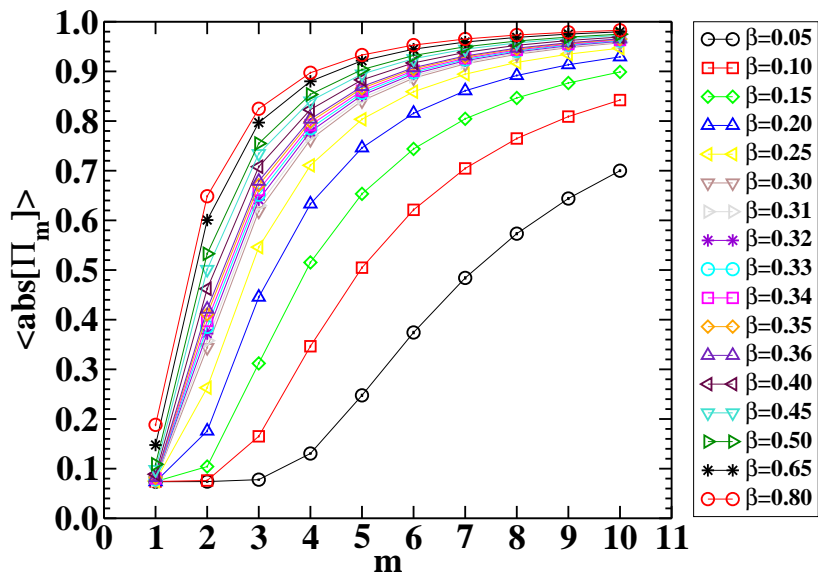


Figure 6: Modulus of $\langle \Pi_m \rangle$ for various values of β versus m on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 5.

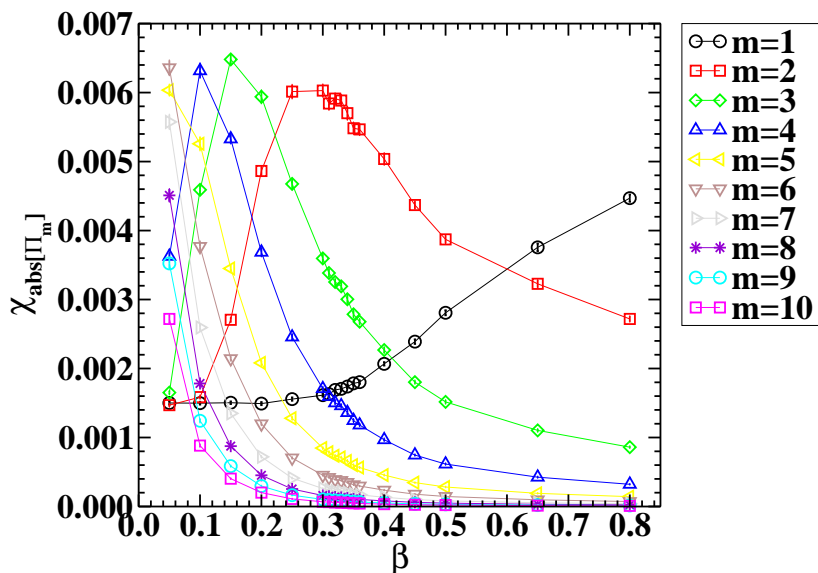


Figure 7: Susceptibility of $\text{abs}[\Pi_m]$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 5.

have done “sliding window” fits, that is we have varied the value of r_{\min} and r_{\max} until obtaining stable values for the fit parameters.

The most important result is that $a\mathcal{M}$ depends neither on the value of m , and so on the fractional charge, nor on the lattice size: $a\mathcal{M} = 0.20 \pm 0.05$. It is instructive to determine the “correlation length” related to this mass: $\xi/a \equiv 1/(a\mathcal{M}) = 5.00 \pm 1.25$; this value, much smaller than the lattice size $\xi/a \ll N_\sigma$, justifies *a posteriori* the fit with an exponential function. We can conclude that this value has a physical meaning: it is the

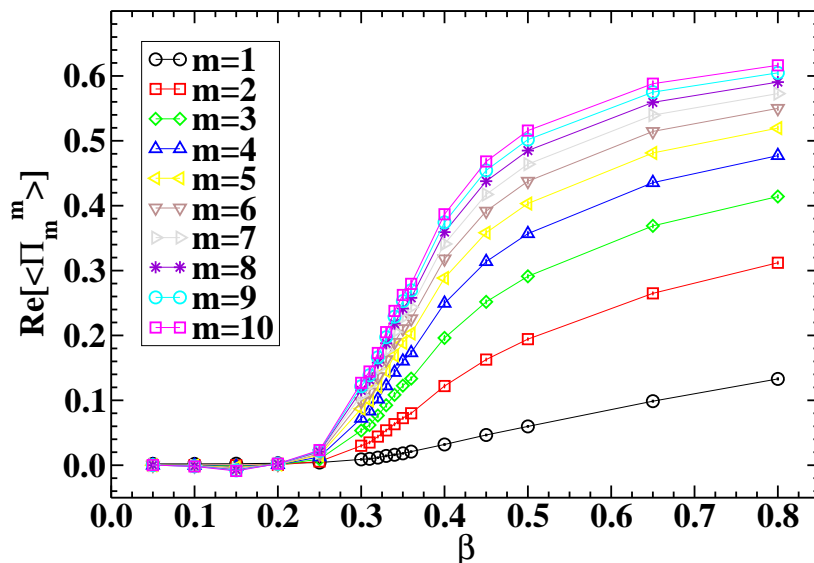


Figure 8: Real part of $\langle \Pi_m^m \rangle$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000.

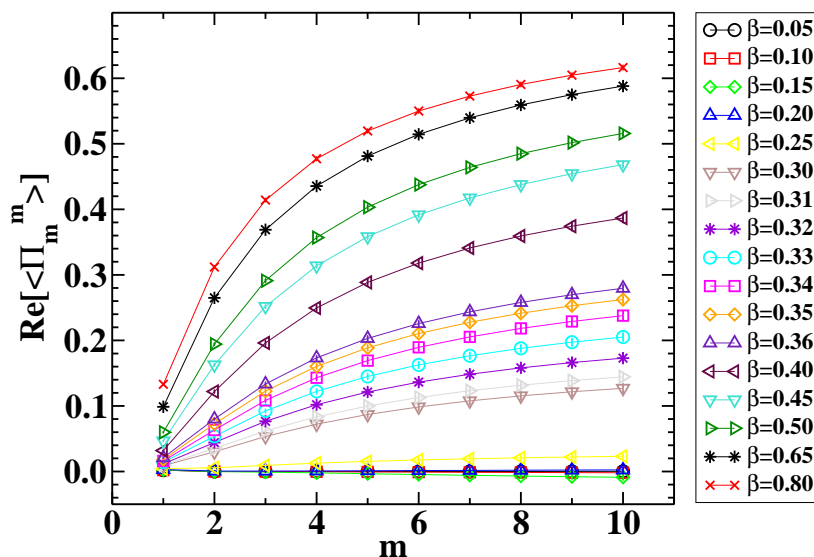


Figure 9: Real part of $\langle \Pi_m^m \rangle$ for various values of β versus m on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 8.

Debye mass characterizing the phase above the critical temperature.

Also for $\beta = 0.25$ we have studied the system on two lattices: $32^2 \times 8$ with $\omega = 0.1$, MCR=50, FOM=1000, NMS=10000, and $64^2 \times 8$ with $\omega = 0.1$, MCR=50, FOM=250, NMS=10000.

As discussed in section 2.2, we expect that here the best fit to the correlator should be given by a power law (see eq. (2.14)); in any case, to be conservative, we consider both the exponential and the power law. Therefore we should interpolate data shown in figure 18

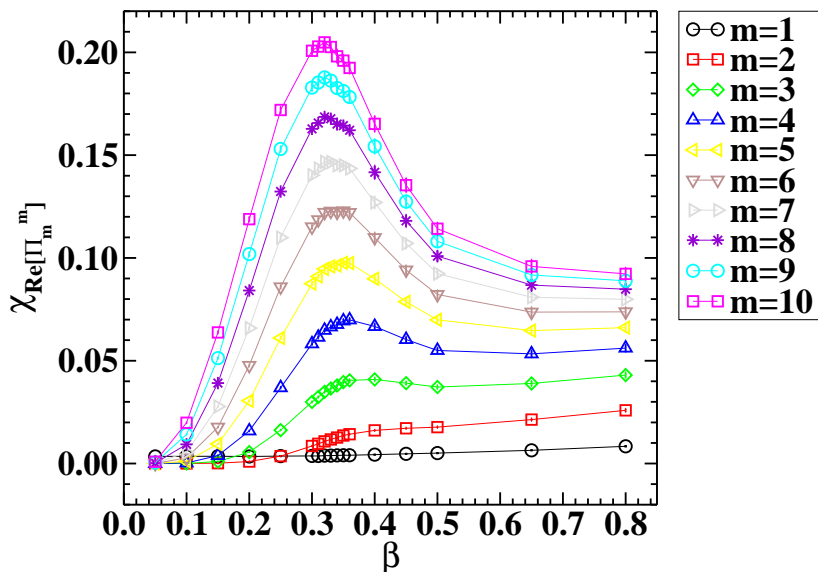


Figure 10: Susceptibility of $\text{Re}[\Pi_m^m]$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 8.

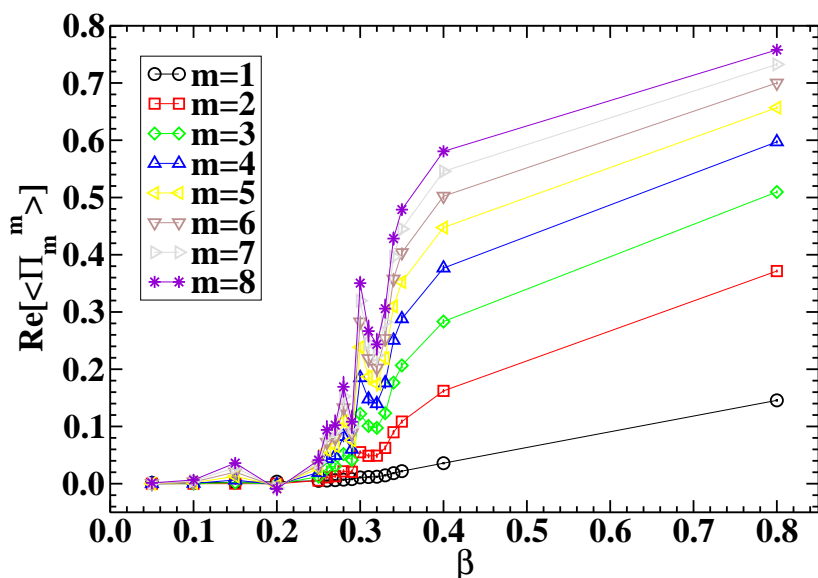


Figure 11: Real part of $\langle \Pi_m^m \rangle$ for 8 values of m versus β on a $32^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are $aM = 0.05$, $\omega = 0.2$, $\delta t = 0.03$, MCR=10, FOM=10, NMS=100000.

with the following two functions:

$$G(r) = A \left(e^{-Mr} + e^{-M(N_\sigma - r)} \right) + C, \quad (4.3)$$

$$G(r) = A \left(r^{-\eta} + (N_\sigma - r)^{-\eta} \right) + C. \quad (4.4)$$

Here the constant C should zero, since the \mathbb{Z} symmetry is unbroken, however, we included it to take into account the possibility that, due to not enough large statistics, the whole

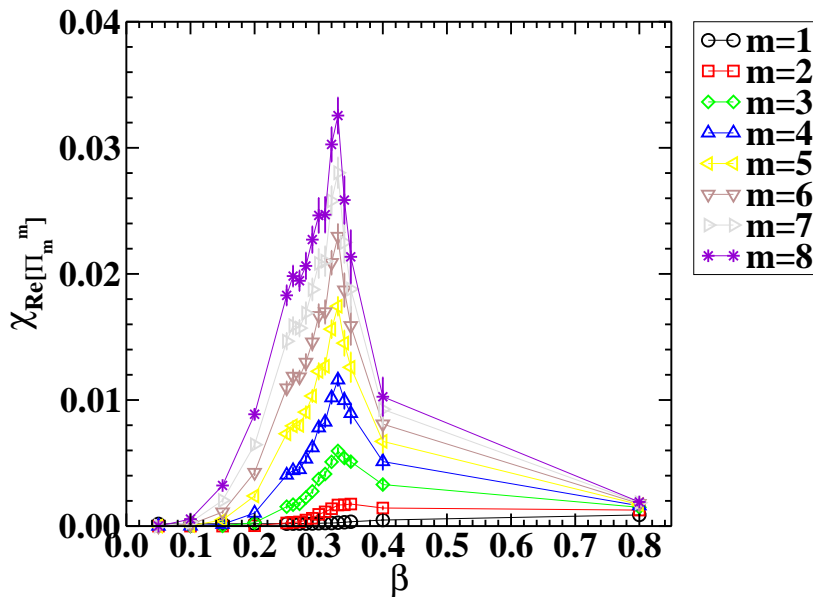


Figure 12: Susceptibility of $\text{Re}[\Pi_m^m]$ for 8 values of m versus β on a $32^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 11.

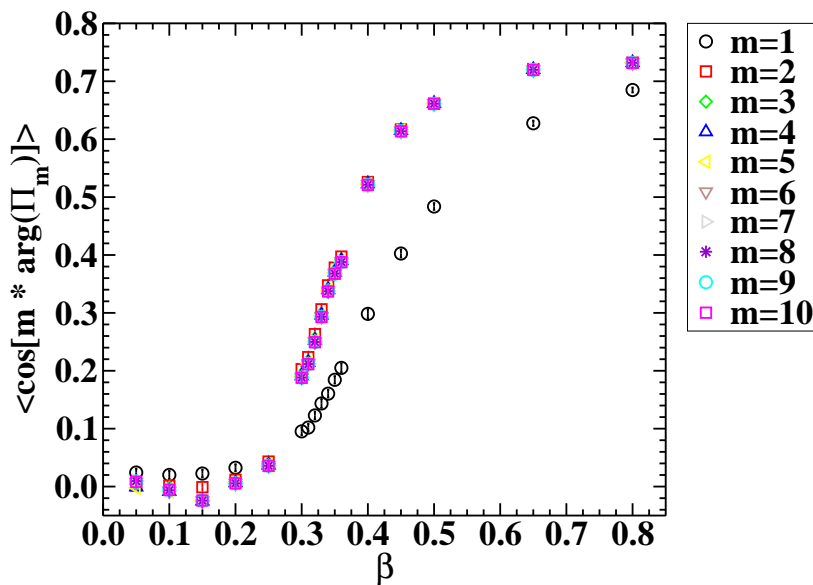


Figure 13: $\langle \cos[m * \arg(\Pi_m)] \rangle$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000.

configuration space is not explored by the simulation algorithm, this leading to $\langle \Pi_m \rangle \neq 0$ (see figure 19). We note, however, that the large distance correlator does not go to zero (see figure 18); this happens because of the small values of \mathcal{M} and η . In this case, if we try to fit data using directly eqs. (4.3) or (4.4), we find that the value of C is not equal to the expected value, i.e. $\text{abs}(\langle \Pi_m \rangle)^2$. Therefore, we first determine the connected correlator, by subtraction of the numerical value of $\text{abs}(\langle \Pi_m \rangle)^2$, and then fit data using eqs. (4.3) or (4.4)

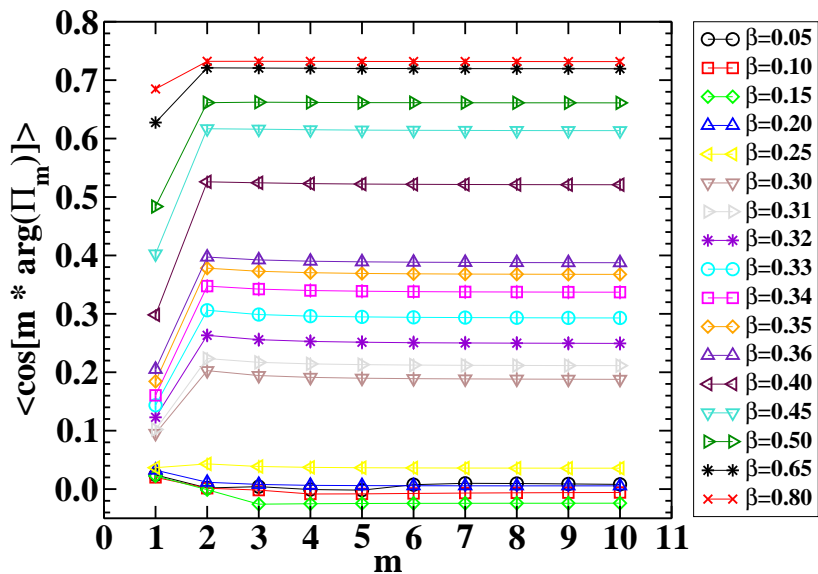


Figure 14: $\langle \cos[m \times \arg \Pi_m] \rangle$ for various values of β versus m on a $12^2 \times 8$ lattice; the solid line is to guide the eye; the simulation parameters are as in figure 13.

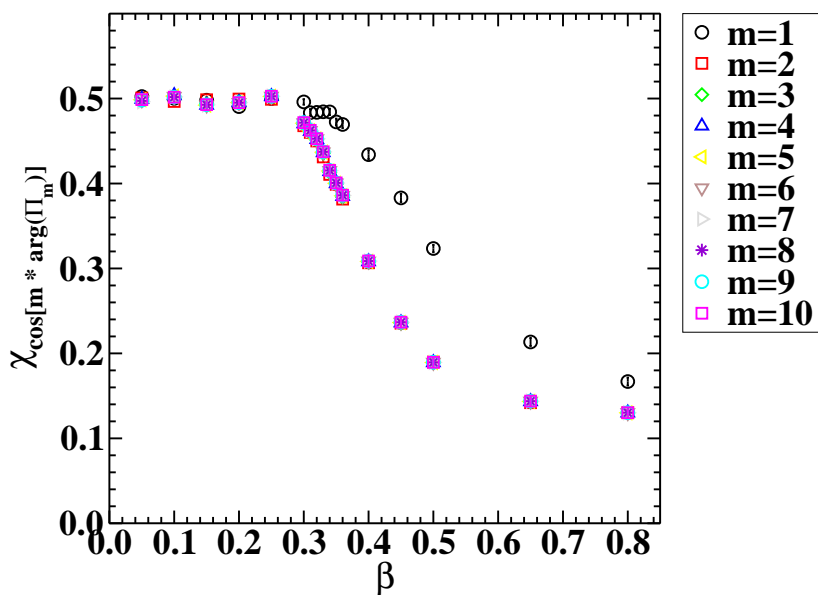


Figure 15: Susceptibility of $\cos[m \times \arg \Pi_m]$ for 10 values of m versus β on a $12^2 \times 8$ lattice; the simulation parameters are as in figure 13.

without the additive constant C :

$$G(r) = A \left(e^{-\mathcal{M}r} + e^{-\mathcal{M}(N_\sigma - r)} \right), \quad (4.5)$$

$$G(r) = A \left(r^{-\eta} + (N_\sigma - r)^{-\eta} \right). \quad (4.6)$$

In figure 20 we show the value of η obtained on the lattice with $N_\sigma = 64$ by a fit with eq. (4.6) for different values of m . Simulations on the lattice with $N_\sigma = 32$ do not permit to obtain reliable estimates, since results are not stable.

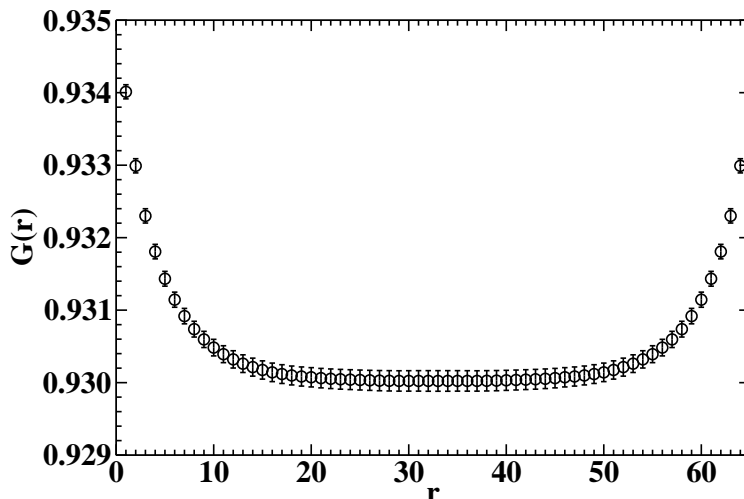


Figure 16: $G(r)$ versus r on a $64^2 \times 8$ lattice at $\beta = 0.40$ (deconfined phase); the simulation parameters are $m = 10$, $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=250, NMS=8000.

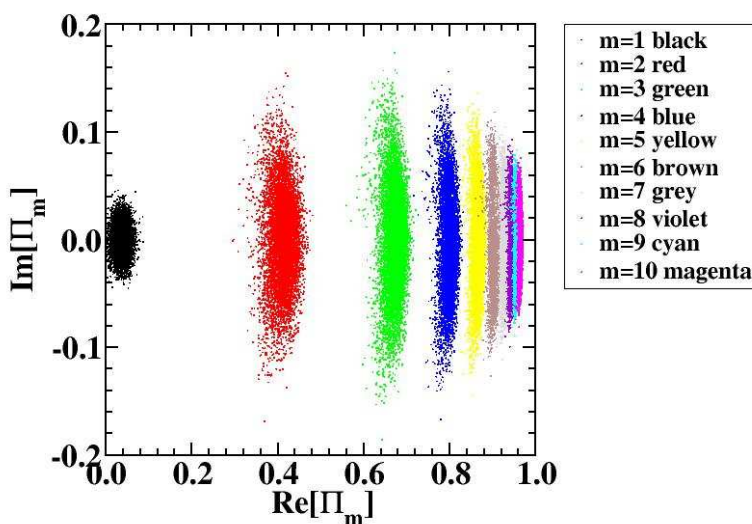


Figure 17: Scatter plots of Π_m for 10 values of m on on a $64^2 \times 8$ lattice at $\beta = 0.40$ (deconfined phase); the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=250, NMS=8000.

It is interesting to note that the behavior of η is the one predicted by eq. (3.13), except for an additive constant, since we find $\eta = A/m^2 + B$ (see figure 21); for the lattice with $N_\sigma = 64$, we find $A = 1.59(64)$ and $B = 0.270(27)$, with $\chi^2/\text{d.o.f.} \sim 0.01$. Note that the tree level theoretical value is $\eta = N_\tau/(m^2 4\pi\beta) = 2.5464\dots/m^2$.

If we try to fit data with eq. (4.5), for both lattices the result is a value of aM that depends weakly from m and it is of the order $1/N_\sigma$; this is a strong evidence that the system is in a critical region with a “correlation length” $\xi/a \propto N_\sigma$, that is for $0 < T < T_c$ the system is always critical. This is a typical feature of the BKT transition.

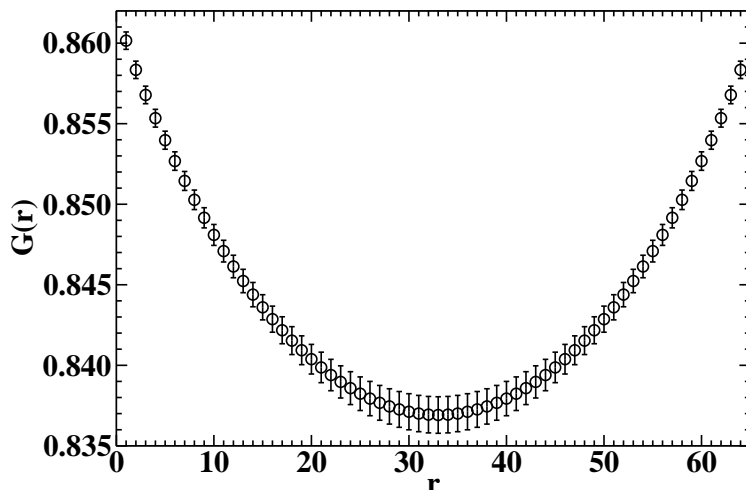


Figure 18: $G(r)$ versus r on a $64^2 \times 8$ lattice at $\beta = 0.25$ (confined phase); the simulation parameters are $m = 10$, $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=250, NMS=10000.

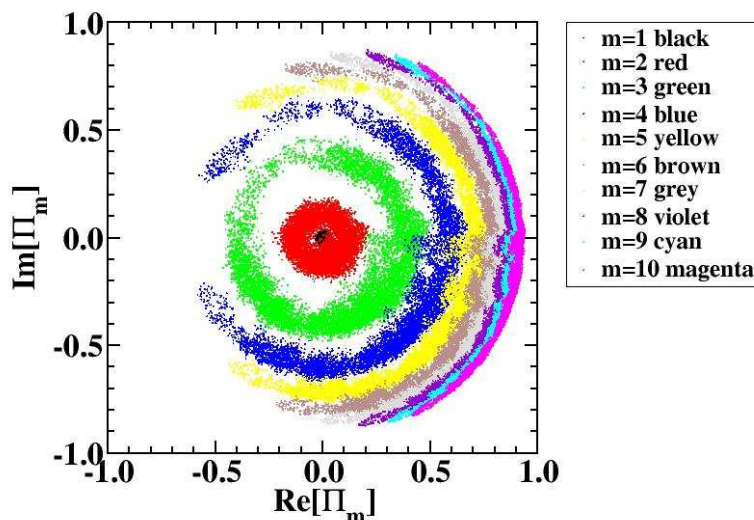


Figure 19: Scatter plots of Π_m for 10 values of m on on a $64^2 \times 8$ lattice at $\beta = 0.25$ (confined phase); the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=250, NMS=10000.

The error bars on aM and η are not those resulting from the fit, which would be underestimated owing to the correlation among the values of $G(r)$ at different r 's. They are estimated, instead, through the behavior of the *effective* η and aM , built from suitable ratios of $G(r)$ in such a way that the dependence on the parameters A and B disappears. This method allowed also to cross-check the results of the fit.

4.4 Chiral condensate

The results on the chiral condensate are presented in figures 22 and 23. Data show a smooth

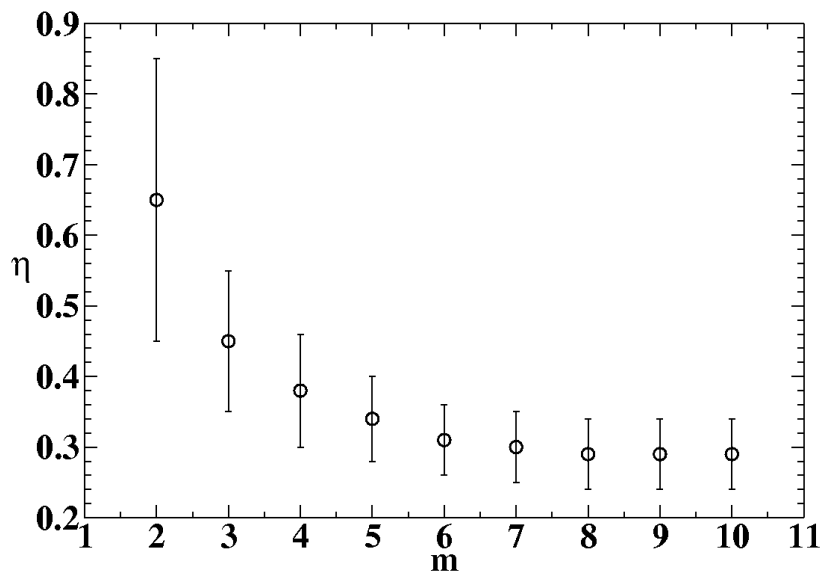


Figure 20: η versus m on a $64^2 \times 8$ lattice at $\beta = 0.25$ (confined phase); the simulation parameters are as in figure 18.

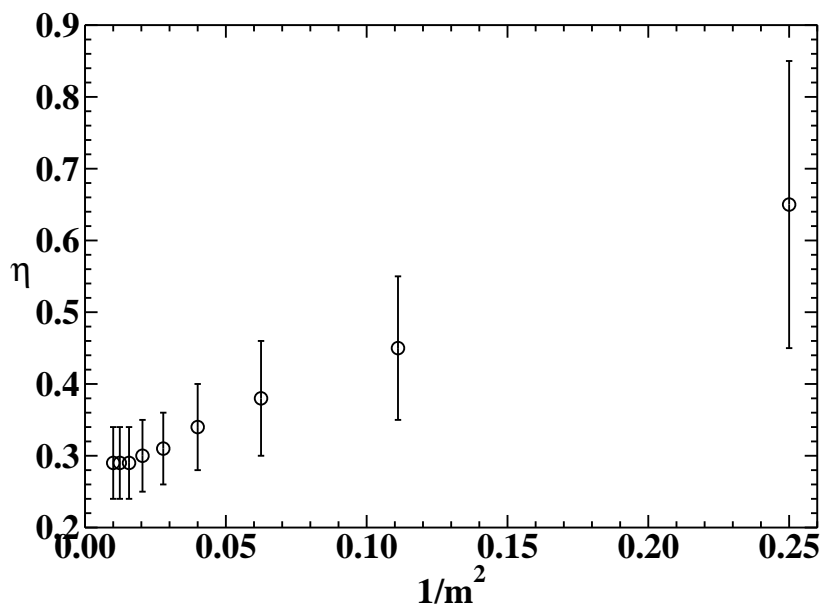


Figure 21: η versus $1/m^2$ on a $64^2 \times 8$ lattice at $\beta = 0.25$ (confined phase); the simulation parameters are as in figure 18.

transition for increasing β , which supports the argument that there is a chiral transition coinciding with the confinement-deconfinement one. Results in favor of this conclusion can be found also in refs. [18, 19].

One would expect, however, that the presence of a transition should be accompanied by a peak in the susceptibility. This seems to be not the case in our analysis — see figure 23. A possible reason for this unexpected result could be the small volume used.

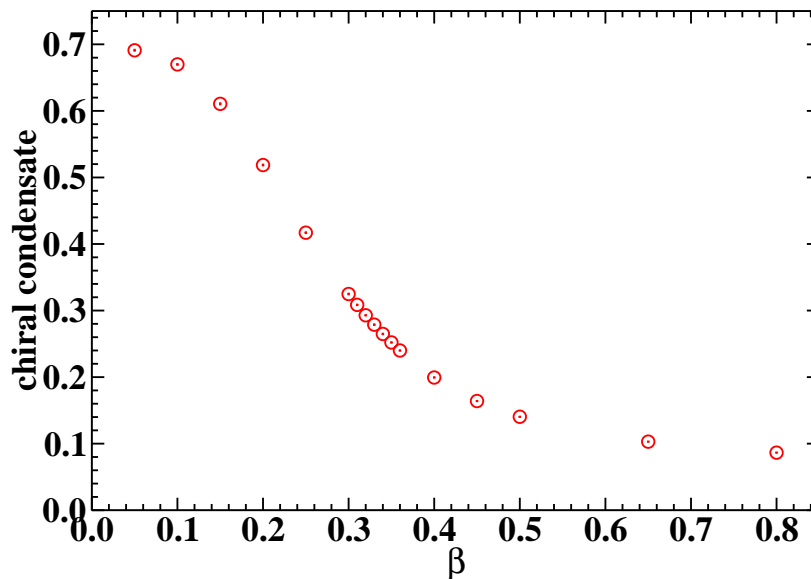


Figure 22: Chiral condensate versus β on a $12^2 \times 8$ lattice; the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000.

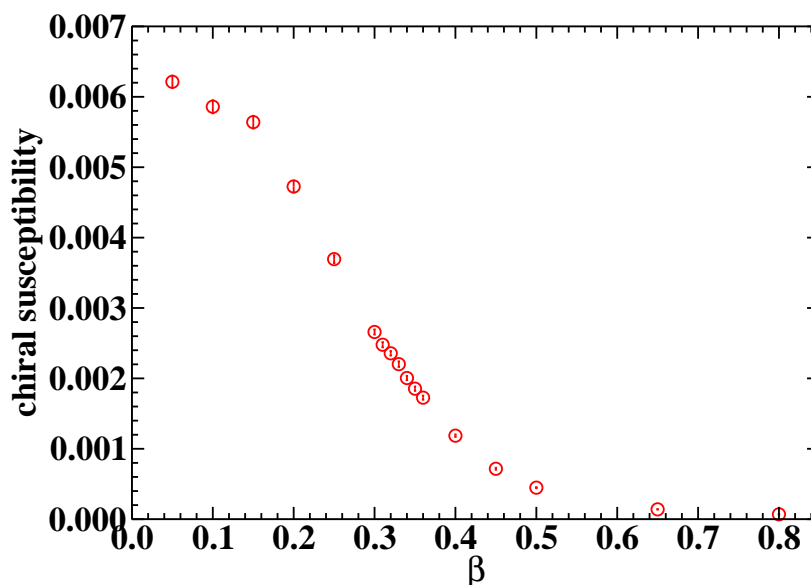


Figure 23: Chiral susceptibility versus β on a $12^2 \times 8$ lattice; the simulation parameters are as in figure 22.

4.5 Monopole density

The behavior of the monopole density and of its susceptibility with β is shown in figures 24 and 25, respectively. The result is somewhat surprising: the monopole density decreases very rapidly in the same region where the Polyakov loop operators show a fast change, but the peak in the susceptibility is located at a different value of the coupling constant, i.e. at $\beta_\rho \simeq 0.18$.

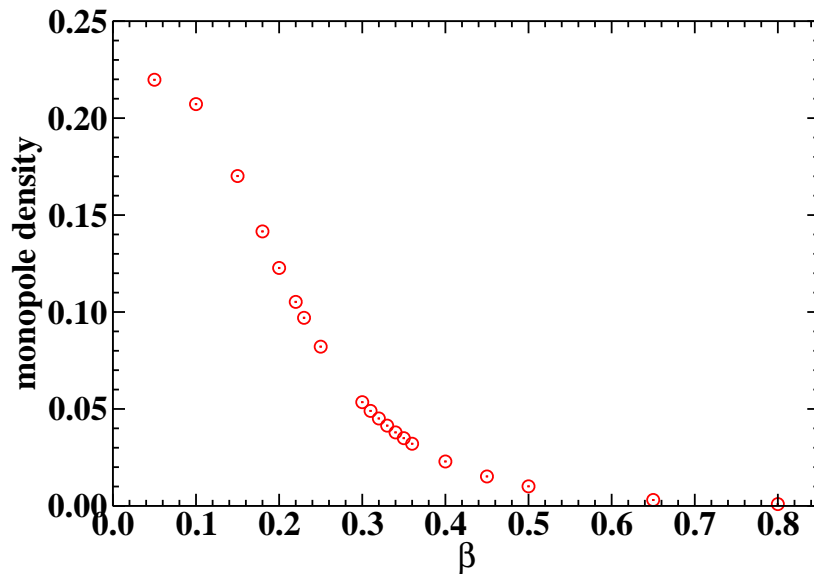


Figure 24: Monopole density versus β on a $12^2 \times 8$ lattice; the simulation parameters are $aM = 0.05$, $\omega = 0.1$, $\delta t = 0.03$, MCR=50, FOM=1000, NMS=10000.

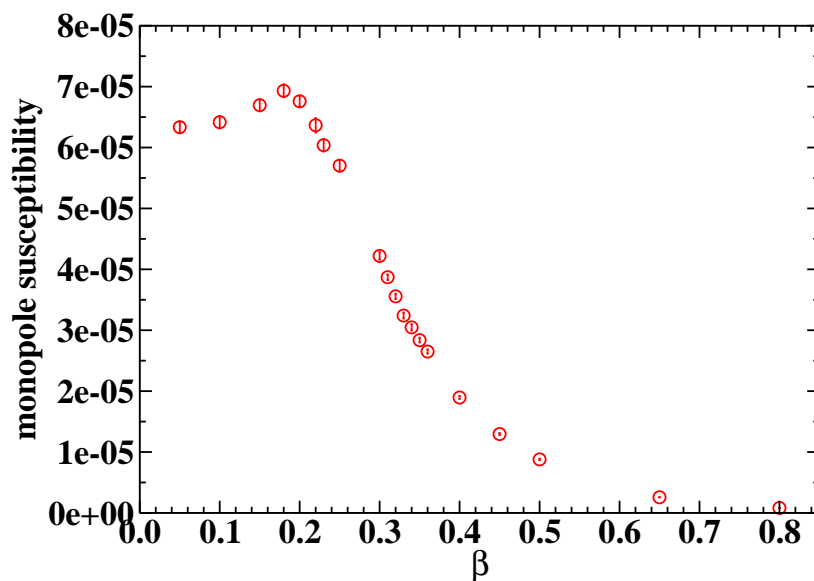


Figure 25: Monopole density susceptibility versus β on a $12^2 \times 8$ lattice; the simulation parameters are as in figure 24.

5. Conclusions and outlook

In this paper we have presented an extended numerical analysis on the confinement-deconfinement transition in non-compact QED₃ with massive fermions at finite temperature.

We have studied the system for a given value of the fermion mass on lattices with different extensions. We have found compelling evidence that there is indeed a transition

temperature from a high-temperature phase where fractional charges are deconfined or Debye-screened and a low-temperature phase with Coulomb-confinement. To detect the transition point we have adopted some suitable lattice operators, sensitive to the breaking of the underlying symmetry of the system, the \mathbb{Z} symmetry. The wall-wall correlation of these operators has permitted to characterize the two phases: the confined one exhibits power law fall-off with the distance, whereas the deconfined one shows exponential decay.

There are several indications that the transition and the phase structure are compatible with the suggestions of refs. [2, 3]:

- the critical temperature found by numerical simulations is in remarkable agreement with the one estimated for large fermion mass in ref. [2] and, in its turn, consistent with the BKT scenario, $T_c = g^2/(8\pi)$; this agreement may be either an indication of smooth mass dependence of the effective action used in ref. [2] or an accidental fact; in both cases, a systematic study of the dependence of our results on the fermionic mass should be performed; this is, however, beyond the scope of the present work;
- two phases are clearly seen, one where the correlator of the order parameter exhibits a fall-off with the distance with a power law, the other where the fall-off is exponential; the fact that the power law is valid well inside the confined phase and not only on the critical point is an indication in favor of the BKT scenario;
- the scaling of the index η for the power law fall-off with the fractional electric charge is in agreement with the prediction from ref. [3] (except for an additive constant), which supports the BKT scenario.

There is an indication that the chiral transition has a relation with the deconfinement transition, which should, however, be confirmed by an analysis on larger lattices.

If one defines monopoles on the lattice in the non-compact theory as in the compact theory, one can see that their density across the deconfinement transition shows a sharp change. Surprisingly enough, the susceptibility of the monopole density shows a peak at a smaller temperature than the deconfinement one.

The future development of this work includes a finite-size scaling analysis near the transition temperature, in order to achieve an accurate enough determination of the critical indices and to conclude that the transition is definitely BKT. Moreover, simulations for different values of the fermion mass could allow to explore the phase diagram of the theory in the space of the mass, temperature and coupling parameters and to get a unifying view of the statistical properties of the system.

Acknowledgments

We thank P. Sodano for drawing our attention to the subject of this work, reading the manuscript and suggesting interesting further developments. We are grateful to D. Giuliano for reading the manuscript and making valuable comments.

References

- [1] N.E. Mavromatos and J. Papavassiliou, *Novel phases and old puzzles in QED3 and related models*, [cond-mat/0311421](#).
- [2] G. Grignani, G.W. Semenoff and P. Sodano, *Confinement-deconfinement transition in three-dimensional QED*, *Phys. Rev. D* **53** (1996) 7157 [[hep-th/9504105](#)].
- [3] G. Grignani, G.W. Semenoff, P. Sodano and O. Tirkkonen, *Charge screening and confinement in the hot 3D-QED*, *Nucl. Phys. B* **473** (1996) 143 [[hep-th/9512048](#)].
- [4] V.L. Berezinsky, *Destruction of long range order in one-dimensional and two-dimensional systems having a continuous symmetry group. 1. Classical systems*, *Sov. Phys. JETP* **32** (1971) 493.
- [5] J.M. Kosterlitz and D.J. Thouless, *Ordering, metastability and phase transitions in two-dimensional*, *J. Phys.* **C6** (1973) 1181.
- [6] A.M. Polyakov, *Quark Confinement and Topology of Gauge Groups*, *Nucl. Phys. B* **120** (1977) 429;
M. Göpfert and G. Mack, *Iterated Mayer expansion for classical gases at low temperatures*, *Commun. Math. Phys.* **81** (1981) 97; *Proof of confinement of static quarks in 3-dimensional U(1) lattice gauge theory for all values of the coupling constant*, *Commun. Math. Phys.* **82** (1982) 545.
- [7] I.F. Herbut and B.H. Seradjeh, *Permanent confinement in the compact QED(3) with fermionic matter*, *Phys. Rev. Lett.* **91** (2003) 171601 [[cond-mat/0305296](#)].
- [8] V. Azcoiti and X.-Q. Luo, *Phase structure of compact lattice QED in three-dimensions with massless Fermions*, *Mod. Phys. Lett. A* **8** (1993) 3635 [[hep-lat/9212011](#)].
- [9] R. Fiore et al., *QED(3) on a space-time lattice: compact versus noncompact formulation*, *Phys. Rev. D* **72** (2005) 094508 [*Erratum ibid.* **72** (2005) 119902] [[hep-lat/0506020](#)].
- [10] S. Hands and R. Wensley, *Effective monopoles in noncompact lattice QED*, *Phys. Rev. Lett.* **63** (1989) 2169.
- [11] P.E.L. Rakow, *Magnetic monopoles in noncompact QED: is there a phase transition?*, *Nucl. Phys.* **30** (*Proc. Suppl.*) (1993) 591 [[hep-lat/9212004](#)].
- [12] S.J. Hands, J.B. Kogut and C.G. Strouthos, *Non-compact QED(3) with $N(f) \geq 2$* , *Nucl. Phys. B* **645** (2002) 321 [[hep-lat/0208030](#)].
- [13] S.A. Gottlieb et al., *The deconfining phase transition and the continuum limit of lattice quantum chromodynamics*, *Phys. Rev. Lett.* **55** (1985) 1958.
- [14] T.A. DeGrand and D. Toussaint, *Topological excitations and Monte Carlo simulation of abelian gauge theory*, *Phys. Rev. D* **22** (1980) 2478.
- [15] S. Duane, A.D. Kennedy, B.J. Pendleton and D. Roweth, *Hybrid Monte Carlo*, *Phys. Lett. B* **195** (1987) 216.
- [16] S. Duane and J.B. Kogut, *The theory of hybrid stochastic algorithms*, *Nucl. Phys. B* **275** (1986) 398.
- [17] A. Kovner and B. Rosenstein, *Kosterlitz-Thouless mechanism of two-dimensional superconductivity*, *Phys. Rev. B* **42** (1990) 4748.

- [18] J.B. Kogut and J.F. Lagae, *Finite temperature QED in three-dimensions with light fermions*, *Nucl. Phys.* **30** (*Proc. Suppl.*) (1993) 737 [[hep-lat/9211059](#)].
- [19] I.J.R. Aitchison and C.D. Fosco, *On the chiral and deconfinement phase transitions in parity-conserving QED(3) at finite temperature*, *Nucl. Phys.* **B 578** (2000) 199 [[hep-th/0002105](#)].